A nonlinear approach to NN interactions using self–interacting meson fields

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Abstract

Motivated by the success of models based on chiral symmetry in NN interactions we investigate self-interacting scalar, pseudoscalar and vector meson fields and their impact for NN forces. We parametrize the corresponding nonlinear field equations and get analytic wavelike solutions. A probability amplitude for the propagation of particle states is calculated and applied in the framework of a boson-exchange NN potential. Using a proper normalization of the meson fields makes all self-scattering amplitudes finite. The same normalization is able to substitute for the phenomenological form factors used in conventional boson exchange potentials and thus yields an phenomenological understanding of this part of the NN interaction. We find an empirical scaling law which relates the meson self-interaction couplings to the pion mass and self-interaction coupling constant. Our model yields np phase shifts comparable to the Bonn B potential results and deuteron properties, in excellent agreement with experimental data.

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I. INTRODUCTION

One of the major problems of today's theoretical nuclear physics is to understand the relationship between the dynamics of the regime where baryons and mesons are observable and the underlying theory, QCD, which contains quark and gluon degrees of freedom. In this context chiral symmetry as a property of the QCD Lagrangian should affect the interaction between hadrons at low and medium energies. There are a number of QCD-inspired hadron models which are based on chiral symmetry [1] and which as a common characteristic imply a nonlinear structure of the meson Lagrangian. Despite their remarkable success in describing qualitative features of hadron interactions, none of these models is able to reach the quantitative agreement with experimental data provided by the conventional boson-exchange (BE) or inversion potentials, which, however, do not have any reference to chiral symmetry [2,3]. Our goal now is to construct a potential model which lies between these extreme positions.

Rather than construct a chiral invariant Lagrangian we want to parametrize the possible effect of chiral symmetry using a phenomenological nonlinear term in the meson Lagrangian which can be interpreted as a meson self-interaction. We solve the corresponding nonlinear field equations analytically and apply the obtained meson propagators in a boson exchange potential for the nucleon-nucleon interaction. Doing so we want to test if and how nonlinearities in the field equations have an influence on *NN* interactions. This is done for each of the meson fields which are used in the Bonn potential model to yield a potential which is able to produce scattering phase shifts and deuteron properties which are in good quantitative agreement with experimental data, comparable well as the Bonn potential [4]. It turns out that the nonlinear effects in the meson fields can give rise to a propagator which not only regularizes the meson self-energy diagrams but in addition can substitute for the empirical form factors used in the Bonn potential. Besides that we find an empirical connection between the phenomenological nonlinearities which ultimately may be traced onto some underlying symmetry.

In our approach we use the model of solitary fields developed by Burt [5]. Here the

decoupled meson field equation is parametrized by

$$\partial_\mu \partial^\mu \Phi + m^2 \Phi + \lambda_1 \Phi^{2p+1} + \lambda_2 \Phi^{4p+1} = 0, \tag{1.1}$$

where Φ is the operator to describe the self-interacting fields. For mesons with nonzero spin this operator is a vector in Minkowski space. The parameter p equals 1/2 or 1 to yield odd or even powered nonlinearities. Using this parametrization, Eq. (1.1) can be solved analytically. In principle there are a lot of different possible types of self-interactions for pseudoscalar, scalar, and vector meson fields. In this first step we will focus on the most simple interaction which agrees with the constraint of parity conservation and Lorentz invariance. This simplification makes the model phenomenological but transparent and easy to handle. What we expect and indeed is confirmed is that it is more important to include nonlinearities at all than to specify their explicit form.

This work is organized as follows. In the next section we introduce the model of solitary meson fields. We will show how Eq. (1.1) can be solved and how the solutions are quantized. After that, in Sec. III, a probability amplitude for the propagation of solitary mesons is defined and evaluated in momentum space. A proper normalization of solitary mesons is depicted in Sec. IV which guarantees that all self-scattering diagrams remain finite. The model then is applied in a boson-exchange potential for NN interactions. No further modifications concerning the meson dynamics will be made. In Sec. V we compare the amplitudes for meson propagation used in the Bonn potential and our model using solitary mesons. It will turn out that it is possible to substitute the Feynman propagator multiplied with the form factors of the Bonn B potential by the solitary meson propagator. Form factors or momentum space cutoff parameters are not needed and additionally we will find an empirical scaling law which relates all parameters of the meson self-interaction to the pion mass and self-interaction coupling constant.

After that we leave the close connection with the Bonn B potential. The scaling law is employed in the one-solitary-boson-exchange potential (OSBEP) which is introduced in Sec. VI. Only the structure of the potential is adopted from the Bonn B OBEP. The meson-

nucleon coupling constants are taken as free parameters and are adjusted to fit np phase shifts and deuteron data. The pion self-interaction coupling constant is also readjusted. All other meson self-interaction coupling constants are then determined by the scaling law. Finally, we will obtain a parameter set which differs from the Bonn B parameters but nevertheless yields a comparable accurate fit to np phase shifts and deuteron properties which are in excellent agreement with experimental data. Together with a discussion of future prospects the results are presented in Secs. VII and VIII. Additionally, np scattering observables have been calculated and will be published elsewhere [6]. For all observables, such as differential cross sections, polarization and spin correlation parameters, the results from OSBEP and Bonn B are comparable in their reproduction of the angular distributions. Furthermore, calculations of pp observables are underway. Here, the inclusion of the Coulomb force requires several modifications with respect to the np potential considered in this work. Besides the change of the nucleon and pion mass a refit of the effective σNN coupling constant is necessary to describe the data. When done, the results will be published together with a χ^2 for the np as well as pp data from the OSBEP with respect to experimental scattering observables.

II. MESON SELF-INTERACTION

A. Solitary mesons

To solve Eq. (1.1) one can make the ansatz for the interacting fields to depend on free fields $\Phi = \Phi(\varphi)$ where φ is a solution of the free Klein-Gordon equation (KGE) with meson mass m:

$$\partial_\mu \partial^\mu \varphi + m^2 \varphi = 0. \tag{2.1}$$

This assumption reduces the nonlinear partial differential equation (1.1) to a nonlinear ordinary differential equation for $\Phi(\varphi)$,

$$\Phi''(\varphi) + \frac{1}{\varphi}\Phi'(\varphi) - \frac{1}{\varphi^2}\Phi(\varphi)$$
$$- \frac{\lambda_1}{m^2\varphi^2}\Phi^{2p+1}(\varphi) - \frac{\lambda_2}{m^2\varphi^2}\Phi^{4p+1}(\varphi) = 0, \tag{2.2}$$

which can be solved by direct integration. The solutions are represented as a power series in φ [5]

$$\Phi = \sum_{n=0}^{\infty} C_n^{1/2p}(w)\, b^n\, \varphi^{2pn+1}. \tag{2.3}$$

These special wavelike solutions of Eq. (1.1) are oscillating functions which propagate with constant shape and velocity. Corresponding to the classical theory of nonlinear waves [7] they shall be called *solitary meson fields*. The coefficients $C_n^a(w)$ are Gegenbauer polynomials [8] defined by

$$\frac{1}{(1 - 2xz + z^2)^a} = \sum_{n=0}^{\infty} C_n^a(x) z^n, \tag{2.4}$$

b and w are functions of the coupling constants and the order p of the self-interaction:

$$b = \left[\left(\frac{\lambda_1}{4(p+1)m^2}\right)^2 - \frac{\lambda_2}{4(2p+1)m^2}\right]^{1/2},$$
$$w = \frac{1}{b}\frac{\lambda_1}{4(p+1)m^2}. \tag{2.5}$$

To quantize the solitary fields we use free wave solutions of Eq. (2.1) in a finite volume V [9],

$$\varphi(x, k, \pm) := \frac{1}{\sqrt{2D_k\omega_k V}}\, a(k, \pm)\, e^{\mp ikx}, \tag{2.6}$$

where the operators $a(k, \pm)$ annihilate quanta of positive or negative energy:

$$\omega_k^2 = \vec{k}^{\,2} + m^2.$$

At this point it is important to notice that we added a factor D_k which is an arbitrary Lorentz invariant function of ω_k. As will become obvious later this constant is crucial for the proper normalization of solitary waves. The creation operators for positive and negative energy quanta are related by

$$a^\dagger(k, +) = a(k, -) \quad \text{and} \quad a^\dagger(k, -) = a(k, +),$$

which imply that it is sufficient to use creation and annihilation operators for positive energy states only. The commutator of these operators reads [10]

$$[a(k), a^{\dagger}(k^{'})] = \delta_{kk'}. \tag{2.7}$$

Using this relation one can construct a complete set of orthonormal eigenvectors to the number operator

$$N_k = a^{\dagger}(k)a(k),$$

which are

$$|N_k\rangle = \frac{1}{\sqrt{N!}}\, {a^{\dagger}}^{N}(k)|0\rangle. \tag{2.8}$$

These are the free N-particle states which form the basis to calculate matrix elements of solitary wave operators. The definition (2.6) determines the solitary meson operator (2.3) and ensures that the projection on the one-particle state

$$|1\rangle\langle 1|\Phi^{\dagger}|0\rangle = \frac{1}{\sqrt{2D_k\omega_k V}}\, a^{\dagger}(k)\, e^{ikx}|0\rangle$$

yields the plane wave solution for a free on-shell meson. The field operator for mesons with spin 1 is the direct product of a Minkowski vector, which describes the spin polarization s, and a Lorentz invariant operator in the Hilbert space of the N-particle states [9]

$$\Phi_v^{\mu}(x, k, s) = \epsilon^{\mu}(k, s)\phi(x, k, s). \tag{2.9}$$

Because of the negative intrinsic parity of vector mesons, the operator $\phi(x, k, s)$ transforms like a pseudoscalar field:

$$\phi \xrightarrow{\mathcal{P}} -\phi.$$

To conserve parity the field equation for self-interacting vector mesons must contain only odd powers of the fields, which restricts p to integer values. Using an expansion of the Proca equation [9] yields

$$\partial_{\nu}\partial^{\nu}\Phi_v^{\mu} + m^2\Phi_v^{\mu} + \lambda_1(\Phi_v^{\nu})^{2p}\Phi_v^{\mu} + \lambda_2(\Phi_v^{\nu})^{4p}\Phi_v^{\mu} = 0. \tag{2.10}$$

The normalization [9]

$$\epsilon_\nu(k,s)\epsilon^\nu(k,s^{'}) = \delta_{ss'}$$

leads to the field equation for the operator ϕ:

$$\partial_\nu\partial^\nu\phi + m^2\phi + \lambda_1\phi^{2p+1} + \lambda_2\phi^{4p+1} = 0.$$

This is the same equation as Eq. (1.1) with integer p. Therefore vector mesons can be described by the product of the polarization vector and a solution (2.3) of the pseudoscalar field equation

$$\Phi_v^\mu(x,k,s) = \epsilon^\mu(k,s)\cdot\Phi_{ps}(x,k,s). \tag{2.11}$$

III. MESON PROPAGATION

Using the solutions (2.3) and the commutator (2.7) for the operators appearing in Eq. (2.6) one can readily calculate matrix elements of solitary field operators between N- and M-particle states. The probability for the propagation of an interacting field can now be defined as the amplitude to create an interacting system at some space-time point x which is annihilated into the vacuum at y. Since the intermediate state is not observable and the particles are not distinguishable, a weighted sum over all intermediate states has to be performed [11]:

$$\begin{aligned} iP(y-x) = \sum_k\sum_{N=0}^\infty \frac{1}{N!}\Big[&\langle 0|\Phi(y,k)|N_k\rangle\langle N_k|\Phi^\dagger(x,k)|0\rangle\theta(y_0-x_0) \\ &+ \langle 0|\Phi(x,k)|N_k\rangle\langle N_k|\Phi^\dagger(y,k)|0\rangle\theta(x_0-y_0)\Big]. \end{aligned} \tag{3.1}$$

A straightforward calculation yields the desired amplitudes in coordinate and momentum space. Defining

$$iP(y-x) = \frac{i}{(2\pi)^4}\int d^4k\ P(k^2,m)\ e^{-ik(y-x)}, \tag{3.2}$$

the momentum space amplitude reads

$$
\begin{aligned}
iP(k^2, m) = &\sum_{n=0}^{\infty} \left[C_n^{1/2p}(w)\right]^2 \\
\times &\frac{b^{2n}}{(2V)^{2pn}} \frac{(2pn+1)^{2pn-2}}{D_k^{2pn+1}(\vec{k}^{\,2} + M_n^2)^{pn}} \, i\Delta_F(k^2, M_n),
\end{aligned} \tag{3.3}
$$

with the Feynman propagator

$$
i\Delta_F(k^2, M_n) = \frac{i}{k_\mu k^\mu - M_n^2}, \tag{3.4}
$$

and a mass-spectrum

$$
M_n = (2pn+1)m.
$$

Since $V\omega_k$ is a Lorentz scalar, the amplitude (3.3) is Lorentz invariant. At this point it is convenient to introduce the dimensionless coupling constants α_1 and α_2 which we define as

$$
\alpha_1 := \frac{\lambda_1}{4(p+1)m^2(2mV)^p},
$$

$$
\alpha_2 := \frac{\lambda_2}{4(2p+1)m^2(2mV)^{2p}}. \tag{3.5}
$$

This yields

$$
w = \frac{\alpha_1}{\sqrt{\alpha_1^2 - \alpha_2}} \tag{3.6}
$$

and

$$
\begin{aligned}
iP(k^2, m) = &\sum_{n=0}^{\infty} \left[C_n^{1/2p}(w)\right]^2 \\
&\times \frac{\left[(m^p\alpha_1)^2 - m^{2p}\alpha_2\right]^n (2pn+1)^{2pn-2}}{D_k^{2pn+1}(\vec{k}^{\,2} + M_n^2)^{pn}} \\
&\times i\Delta_F(k^2, M_n).
\end{aligned} \tag{3.7}
$$

The amplitude (3.7) shall be referred to as *modified solitary meson propagator*. For $p = 1$ one gets the amplitude for the propagation of pseudoscalar fields and $p = 1/2$ describes scalar particles. The functional form, i. e. the mass spectrum and the momentum dependence, is dominantly determined by the order p of the self-interaction rather than by the coupling constants λ_1 and λ_2. To keep the model simple, we restrict our considerations to the case

$\lambda_2 = 0$. The series (3.7) converges rapidly, depending on the mass, and the subsequent terms diminish by two (π) or three (ω) orders of magnitude; in practical calculations it is sufficient to use $n_{\max} = 4$.

The amplitude for vector mesons requires an additional sum over the possible polarizations

$$iP_v^{\mu\nu}(y-x) = \sum_{k,s,N} \frac{1}{N!}\Big[\langle 0|\Phi_v^{\mu}(y,k,s)|N_k\rangle\langle N_k|\Phi_v^{\nu\dagger}(x,k,s)|0\rangle\theta(y_0-x_0) + \langle 0|\Phi_v^{\mu}(x,k,s)|N_k\rangle\langle N_k|\Phi_v^{\nu\dagger}(y,k,s)|0\rangle\theta(x_0-y_0)\Big].$$

Using the representation (2.11) and the expression for the sum over the polarizations [9],

$$\sum_s \epsilon^{\mu}(k,s)\epsilon^{\nu}(k,s) = -g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{m_v^2}, \tag{3.8}$$

the momentum space propagator is found to be

$$iP_v^{\mu\nu}(k^2,m_v) = \sum_n \left(-g^{\mu\nu} + \frac{k^{\mu}k^{\nu}}{M_{n,v}^2}\right)(m_v\alpha_v)^{2n} \times \frac{(2n+1)^{2n-2}}{D_{k,v}^{2n+1}\left(\vec{k}^{\,2}+M_{n,v}^2\right)^n}\, i\Delta_F(k^2,M_{n,v}), \tag{3.9}$$

where $iP_{ps}(k^2,m_v)$ equals the propagator (3.7) with integer p and $M_{n,v} = (2n+1)m_v$. For application in NN scattering, one drops the part proportional to $k^{\mu}k^{\nu}$ and the propagator simplifies to

$$iP_v^{\mu\nu}(k^2,m_v) = -g^{\mu\nu} iP_{ps}(k^2,m_v).$$

IV. PROPER NORMALIZATION

The propagator (3.7) contains the arbitrary constant D_k. This constant can depend on the energy ω_k and the coupling constants and is fixed by the following conditions [5]: (i) all amplitudes must be Lorentz invariant, (ii) D_k must be dimensionless, (iii) all self-scattering diagrams must be finite, (iv) the fields have to vanish for $\lambda_1, \lambda_2 \to 0$. Whereas the first three conditions are evident, the last one requires us to recall that, if a particle has no interaction

then there is no way to create or measure it and the amplitude for such a process vanishes; the field exists solely because of its interaction.

The proper normalization constant is a powerful tool to avoid the problem of regularization which arises in conventional models. In a $\lambda\Phi^4$ theory, for example, which is described by setting $\lambda_2 = 0$ and $p = 1$, one gets infinite results calculating the first correction to the two-point function $iP(y-x)$. A proper normalization, i. e., using the smallest power

$$D_k \sim (\omega_k V)^2,$$

makes the result finite. A different situation occurs in models including massive spin-1 bosons. Such a case, with or without self-interaction, is harder to regularize due to the additional momentum dependence in the Minkowski tensor which appears in Eq. (3.9). Nevertheless, the vector mesons ρ and ω are important ingredients in every boson-exchange model. A minimum power proper normalization to solve this problem is

$$D_k \sim (\omega_k V)^4.$$

In summary, we satisfy the above stated four conditions with

$$D_k = \left\{ 1 + \left[\left(\frac{m^2}{\lambda_1}\right)^{2/p} + \left(\frac{m^2}{\lambda_2}\right)^{1/p} \right] (\omega_k V)^2 \right\}^{\kappa} \tag{4.1}$$

$$\text{where} \quad \begin{cases} \kappa = 1 & \text{for scalar and} \\ & \text{pseudoscalar mesons,} \\ \kappa = 2 & \text{for vector mesons.} \end{cases}$$

For later application in *NN* potentials we will consider the interaction where $\lambda_2 = 0$ and $p = 1/2$ or $p = 1$ for scalar or pseudoscalar and vector mesons, respectively. Again, we will use the dimensionless coupling constants (3.5) ($\alpha = \alpha_1$) to get the following amplitudes

(i) Scalar mesons:

$$\begin{aligned} iP_s(k^2, m_s) = &\sum_{n=0}^{\infty} (\sqrt{m_s}\alpha_s)^{2n} \\ &\times \frac{(n+1)^n}{D_{k,s}^{n+1} \left(\vec{k}^{\,2} + M_{n,s}^2\right)^{\frac{n}{2}}} \, i\Delta_F(k^2, M_{n,s}), \end{aligned} \tag{4.2}$$

with $M_{n,s} = (n+1)m_s$ and

$$D_{k,s} = 1 + \left(\frac{m^2}{\lambda_s}\right)^4 (\omega_k V)^2$$
$$= 1 + \frac{1}{5184(\sqrt{m_s}\alpha_s)^4} \left(\frac{\vec{k}^{\,2}}{(n+1)^2} + m_s^2\right). \tag{4.3}$$

(ii) Pseudoscalar mesons:

$$iP_{ps}(k^2, m_{ps}) = \sum_n (m_{ps}\alpha_{ps})^{2n}$$
$$\times \frac{(2n+1)^{2n-2}}{D_{k,ps}^{2n+1}\left(\vec{k}^{\,2} + M_{n,ps}^2\right)^n}\, i\Delta_F(k^2, M_{n,ps}), \tag{4.4}$$

with $M_{n,ps} = (2n+1)m_{ps}$ and

$$D_{k,ps} = 1 + \left(\frac{m^2}{\lambda_{ps}}\right)^2 (\omega_k V)^2$$
$$= 1 + \frac{1}{256(m_{ps}\alpha_{ps})^2} \left(\frac{\vec{k}^{\,2}}{(2n+1)^2} + m_{ps}^2\right). \tag{4.5}$$

(iii) Vector mesons:

$$iP_v^{\mu\nu}(k^2, m_v) = \sum_n \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{M_{n,v}^2}\right)(m_v\alpha_v)^{2n}$$
$$\times \frac{(2n+1)^{2n-2}}{D_{k,v}^{2n+1}\left(\vec{k}^{\,2} + M_{n,v}^2\right)^n}\, i\Delta_F(k^2, M_{n,v}), \tag{4.6}$$

with $M_{n,v} = (2n+1)m_v$ and

$$D_{k,v} = \left[1 + \left(\frac{m^2}{\lambda_v}\right)^2 (\omega_k V)^2\right]^2$$
$$= \left[1 + \frac{1}{256(m_v\alpha_v)^2} \left(\frac{\vec{k}^{\,2}}{(2n+1)^2} + m_v^2\right)\right]^2. \tag{4.7}$$

V. PROPAGATOR AND *NN* POTENTIALS

With its proper normalization the modified solitary meson propagator is now completely determined. Without any modification we next apply the model of solitary mesons in a

boson-exchange potential for NN interactions. As guiding line we use the Bonn potential [4]. In this model different sets of parameters allow a good quantitative description of NN scattering data as well as deuteron and nuclear matter properties. Nevertheless, these potentials contain phenomenological cutoff masses entering the form factors applied at each meson-nucleon vertex. The cutoff masses are free parameters and each of them is adjusted to fit the experimental data.

As a special example we regard the Bonn B OBEP [12] since this parametrization of the Bonn potential is easy to handle and provides good agreement with experiment. The Bonn B potential takes into account the contributions of mesons with masses below 1 GeV. These are the scalar δ, the pseudoscalar π and η, and the vector mesons ρ and ω. Additionally one includes the virtual scalar σ_0 and σ_1 in isospin $T = 0$ and $T = 1$ channels, respectively, to simulate correlated two-pion exchange [4].

In our approach we will adopt all features concerning nucleon kinematics and the structure of meson-nucleon coupling from the Bonn B potential. The important difference will be the treatment of the meson dynamics. In the Bonn B model a renormalized Feynman propagator is used for the mesons in intermediate states. Additionally one has to attach a form factor to each meson-nucleon vertex to achieve convergence of the momentum space potentials. Combined one gets the amplitude for the exchange of meson $\beta = \pi,...,\delta$,

$$\left[F_\beta(k^2)\right]^2 i\Delta_F(k^2, m_\beta) \tag{5.1}$$

with

$$F_\beta(k^2) = \left(\frac{\Lambda_\beta^2 - m_\beta^2}{\Lambda_\beta^2 - k^2}\right)^{n_\beta} .$$

For vector mesons this amplitude is multiplied with the tensor (3.8). The values of the Bonn B parameter set for the cutoff masses and the exponent n_β can be found in Table I.

In a model with solitary bosons as excess particles the modified solitary meson propagator (3.7) should be used instead of the free Feynman propagator. The question arises if form factors are still necessary. In this context one must remember that, due to the proper normalization, the solitary meson amplitude already contains a strong decay with increasing

momentum. It is important to emphasize that this normalization was done independently of the application in NN interactions. Therefore if the amplitude (5.1) is replaced by the modified solitary meson propagator with proper normalization we may be able to solve two problems of a boson-exchange potential simultaneously: (i) regularization of self-energy amplitudes and (ii) convergence of momentum space potentials. Form factors are not necessary any more and the propagator (3.7) is all that is needed to describe the meson dynamics.

In the Bonn potential, form factors are used in a sophisticated but empirical manner. Replacing this phenomenology by effects of nonlinearities in the meson field equations we conjecture new aspects in this part of the boson-exchange model.

To test the validity of this substitution we first compare the Feynman propagator multiplied with the form factor of the Bonn potential to the solitary meson propagators (4.2-4.6) before we calculate a solitary boson exchange potential.

To make this comparison we recall that we use the Blankenbecler-Sugar (BbS) reduction [13] of the Bethe-Salpeter (BS) equation [14]. Here the relativistic two-nucleon propagator is replaced by the BbS propagator

$$g(k,s) = \delta(k_0)\,\frac{M^2}{E_k}\,\frac{\Lambda_+^{(1)}(\vec{k})\Lambda_+^{(2)}(-\vec{k})}{\frac{1}{4}s - E_k^2 + i\epsilon},$$

where $\sqrt{s}$ is the total energy in the center-of-mass frame, k is the momentum of the intermediate meson, M the nucleon mass and $\Lambda_+^{(i)}$ is the positive energy projection operator for nucleon i. As a result of the delta-function, the matrix element of NN scattering is applied at $k_0 = 0$. The amplitudes describing meson propagation therefore depend on $\vec{k}^{\,2}$ only and can easily be compared.

In Figs. 1 and 2 we show the Bonn B amplitudes (5.1) for the different mesons compared with the solitary meson propagators. Using the parameters of the Bonn B potential one can always find appropriate values of the self-interaction coupling constant α_β to make the solitary wave propagator fit the Feynman propagator multiplied by the form factor. Obviously the solitary meson propagator yields the same amplitude as the Feynman propagator dressed with the meson-nucleon form factor. This confirms our anticipation that inclusion

of a more elaborated meson dynamics might be able to substitute for the form factor.

The coupling constants which were found by fitting the solitary meson propagators to the amplitudes (5.1) are listed in Table II. It is obvious that they decrease with increasing mass which implies a dependence $\alpha(m)$. In fact, looking at the propagators (4.2)-(4.6) one finds that the mass and the coupling constant always appear in the combination $\sqrt{m}\alpha$ for scalar and $m\alpha$ for pseudoscalar and vector fields. From this one can guess an empirical scaling law which reads

$$\begin{aligned} \alpha(m) &= \alpha_\pi \cdot \left(\frac{m_\pi}{m}\right)^{\frac{1}{2}} && \text{for scalar fields,} \\ \frac{\alpha(m)}{\sqrt{\kappa}} &= \frac{\alpha_\pi}{\sqrt{\kappa_\pi}} \cdot \left(\frac{m_\pi}{m}\right) && \text{for pseudoscalar and vector fields.} \end{aligned} \tag{5.2}$$

To test these relations we took the pion self-interaction coupling constant $\alpha_\pi = 0.36$ and calculated all other coupling constants using the scaling law (5.2). In Fig. 3 we show the predicted mass dependence for fields with quadratic and cubic self-interactions compared to the coupling constants from Table II obtained by fitting the propagator (3.7) to the Bonn B amplitudes. The agreement for both kinds of self-interactions is amazing. Consequently, we conclude that the modified solitary meson propagator is not only an equivalent description for the meson exchange amplitude as the Feynman propagator combined with a form factor but additionally yields a connection between the meson parameters by a simple scaling law. This might be the residue of some underlying symmetry and therefore puts some physical significance to the parameter α_π.

The deviations from the predicted mass dependence are understandable since the cutoff masses of the Bonn B potential are free and uncorrelated parameters. Thus their values can be readjusted in a global fit together with the meson-nucleon coupling constants, enforcing the self-interaction coupling constants to agree with the scaling law.

In the next section we will calculate a boson-exchange potential using self-interacting mesons as exchange particles. From the beginning we will employ the scaling law (5.2)

and therefore the pion self-interaction coupling constant will be the only parameter for the dynamics of all mesons.

VI. ONE-SOLITARY-BOSON-EXCHANGE POTENTIAL

In the former section we have shown the equivalence between describing the meson exchange amplitude by a Feynman propagator multiplied with a form factor and the usage of the modified solitary meson propagators (4.2)-(4.6). With this identification it is straight forward to calculate a momentum space potential for elastic NN scattering. We take the Bonn B potential and replace the amplitude (5.1) by the propagator (3.7). All other features remain essentially unchanged. A detailed description of the potential can be found in the genuine publication by Machleidt, Holinde, and Elster [4]; for further details, see Refs. [12,15].

Since the conventional terminology of form factors is abandoned in the solitary boson-exchange potential, we do not have any cutoff parameters in our model. We strictly use the scaling law for the meson self-interaction. Besides the meson-nucleon coupling constants the pion self-interaction coupling constant is the only adjustable parameter; all other meson parameters are determined by the scaling law (5.2). After adjusting the free parameters to np data we obtain a parameter set which differs from the Bonn B parameters, implying that the solitary meson propagators no longer fit the Bonn B amplitudes. However, the results in np interactions are in very good agreement with experimental data, as we show in the next section.

VII. RESULTS

To test our model we calculated np phase shifts for total angular momenta $J \leq 3$ and deuteron properties, neglecting meson-exchange-current contributions. Besides the meson masses in Table I the values for the nucleon mass and $\hbar c$ are the same as in the Bonn

potential [4]

$$M = 938.926 \text{ MeV} \quad \text{and} \quad \hbar c = 197.3286 \text{ MeV fm}.$$

Most important we consistently applied the scaling law (5.2) for the meson self-interaction coupling constants. Thus the only adjustable parameters in the OSBEP are the pion self-interaction coupling constant α_π and the meson-nucleon coupling constants. For the πNN coupling we used the experimentally fixed value

$$\frac{g_\pi^2}{4\pi} = 13.7,$$

which was determined by the πN and NN phase shift analysis by Arndt *et al.* [16]. After all, our model includes eight parameters which are adjusted to experiment. As experimental input the VPI-SM95 single energy phase shift analysis [17] and phases from the Nijmegen partial wave analysis (PWA) [18] are available. Our model parameters were solely fitted to the SM95 phase shifts which, in contrast to the Nijmegen PWA, are given at single energies with individual error bars. This allows one to calculate a measure of distance (χ^2) for the model phase shifts with respect to the phases obtained from experiment. Table III contains the parameter set which produced the best agreement between model predictions and experimental data. Together with the Bonn B phase shifts [12] the phase shifts from the OSBEP are shown in Figs. 4 and 5. Experimental values for the deuteron properties were taken from different sources, they are shown in Table IV compared with our theoretical results.

VIII. CONCLUSIONS AND OUTLOOK

To compare the OSBEP and Bonn B results we took the SM95 phase shifts and calculated (i) the distance to the OSBEP, (ii) the distance to the Bonn B potential and (iii) the distance to the Nijmegen PWA. The respective numbers (χ^2/datum) are 64.2 for the OSBEP, 92.2 for the Bonn B potential and 19.8 for the Nijmegen phase shifts. The slightly larger value for the Bonn B potential arises since the parameters therein were adjusted to the phase

shifts available in the 1980s. Since the Nijmegen PWA yields a χ^2/datum with respect to the scattering observables which is of order 1 (in their weighting of data), the distance between Nijmegen PWA and SM95 is a measure of consistency for the available phase shifts. If we divide the distances of the OSBEP and the Bonn B potential by this number we obtain a measure of consistency between the theoretical and experimental data. The respective numbers are 3.2 for the OSBEP and 4.7 for the Bonn B phase shifts. On this basis, the agreement between OSBEP and experiment can be considered as excellent, at least comparable to the Bonn B results. Additionally, the deuteron properties listed in Table IV coincide very well with the experimental data. These results were achieved by using eight parameters only which enforces the evidence for the scaling law (5.2).

We incorporated the nonlinear character of chiral meson dynamics in a phenomenological way. Doing so the effect was not only the possibility to substitute the form factors of conventional models by a solitary meson propagator but additionally we found an empirical scaling law to unify the meson self-interactions using a single constant α_π only. The advantage of this result is a more consistent description of the meson-nucleon coupling. Because of the absence of form factors, the Ward identities for the conserved nucleon currents are satisfied at each meson-nucleon vertex, a circumstance which is not the case in the Bonn B potential. The scaling law reflects a connection between the mesons which may be traced to some fundamental symmetry on the quark sector and therefore gives the parameter α_π a strong physical significance. Knowing the astonishing influence of the meson self-interaction in NN interactions motivates a more detailed investigation concerning the connection between chiral symmetry and the nonlinear effects shown here.

Clearly the model is a step towards a chiral symmetric boson-exchange model but there is still a long way to go. First of all a realistic chiral invariant Lagrangian must be constructed which contains all the mesonic degrees of freedom of the boson-exchange model. The mechanism of spontaneous symmetry breaking has to be worked out to explain the connection between the meson parameters described by the scaling law. The possibility to find such a Lagrangian, which yields excellent results in NN scattering by construction when

related to the nonlinear field equations used here, is an exciting challenge for future work.

For nuclear reaction and nuclear structure calculations the solitary-boson-exchange potential can be applied in the same way as the Bonn B potential. Furthermore, we expect that especially πN scattering or pion production processes calculated in the framework of a model with solitary mesons may show interesting results. Whereas the higher order terms ($n > 0$) in the solitary pion propagator (4.4) have only a small influence in NN interactions they can produce a major effect in pion production where momenta are small and heavy mesons are kinematically suppressed.

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TABLES

TABLE I. Bonn B meson parameters [12].

	π	η	ρ	ω	σ_1	σ_0	δ
S^P	0^-	0^-	1^-	1^-	0^+	0^+	0^+
m_β (MeV)	138.03	548.8	769	782.6	550	720	983
Λ_β (GeV)	1.7	1.5	1.85	1.85	1.9	2.0	2.0
n_β	1	1	2	2	1	1	1

TABLE II. Parameters from fitting the solitary meson propagator to the Bonn B amplitude.

	π	η	ρ	ω	σ_1	σ_0	δ
S^P	0^-	0^-	1^-	1^-	0^+	0^+	0^+
m_β	138.03	548.8	769	782.6	550	720	983
α_β	0.36	0.1079	0.091	0.090	0.1755	0.151	0.1269

TABLE III. OSBEP parameters.

	π	η	ρ	ω	σ_0	σ_1	δ
S^P	0^-	0^-	1^-	1^-	0^+	0^+	0^+
$\dfrac{g_\beta^2}{4\pi}$	13.7	1.398	1.140	18.71	14.15	7.8389	1.369
	$\alpha_\pi = 0.428321$				$f_\rho/g_\rho = 4.422$		

TABLE IV. Deuteron Properties

	Bonn B [12]	OSBEP	Exp.	Ref.
E_B (MeV)	2.2246	2.224590	2.22458900(22)	[19]
μ_d	0.8514 [a]	0.8532 [a]	0.857406(1)	[20]
Q_d (fm^2)	0.2783 [a]	0.2670 [a]	0.2859(3)	[21]
A_S ($\mathrm{fm}^{-1/2}$)	0.8860	0.8792	0.8802(20)	[21]
D/S	0.0264	0.0256	0.0256(4)	[22]
r_{RMS} (fm)	1.9688	1.9539	1.9627(38)	[21]
P_D (%)	4.99	4.6528	–	–

[a]Meson exchange current contributions not included

FIGURES

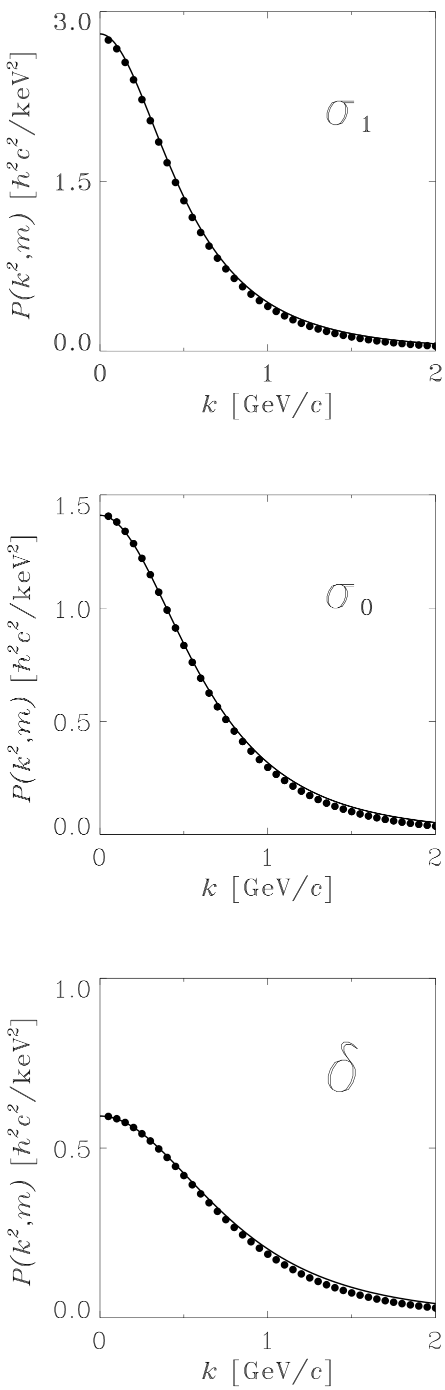

FIG. 1. Solitary meson propagators for quadratic self-interaction (full) compared to the corresponding Feynman-propagators multiplied with the Bonn B form factors (circles).

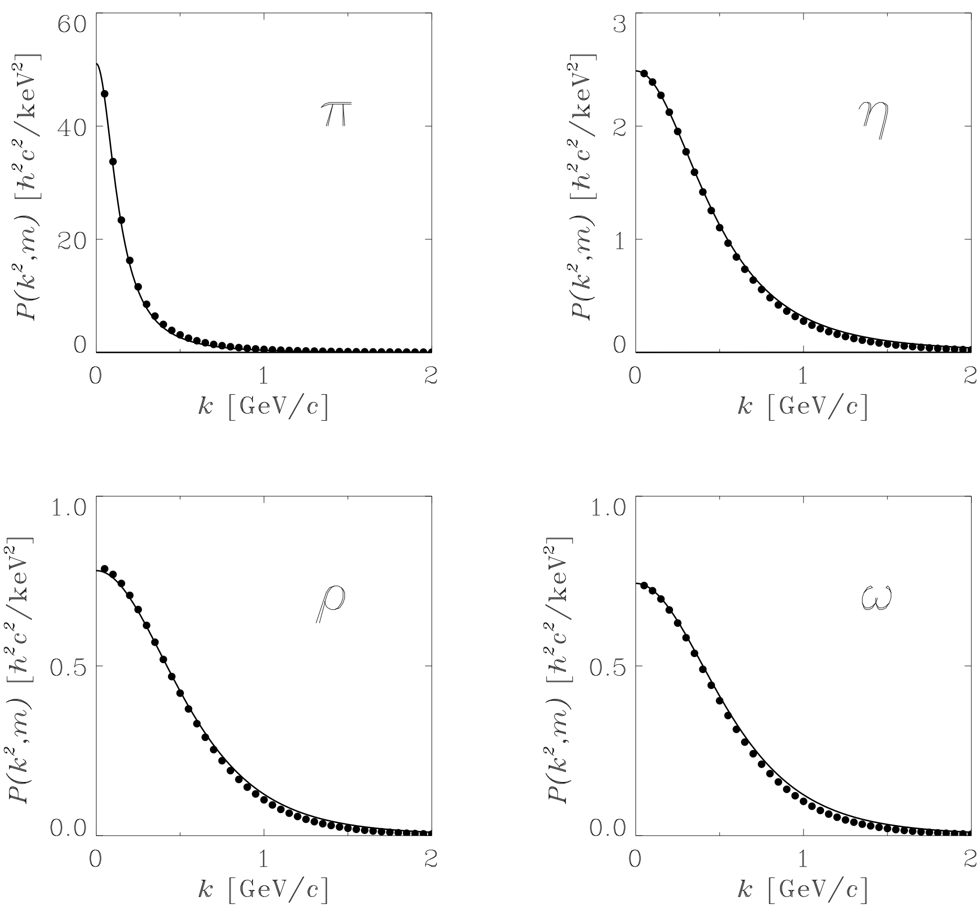

FIG. 2. Solitary meson propagators for cubic self-int. (full) versus Bonn B amplitudes (circles).

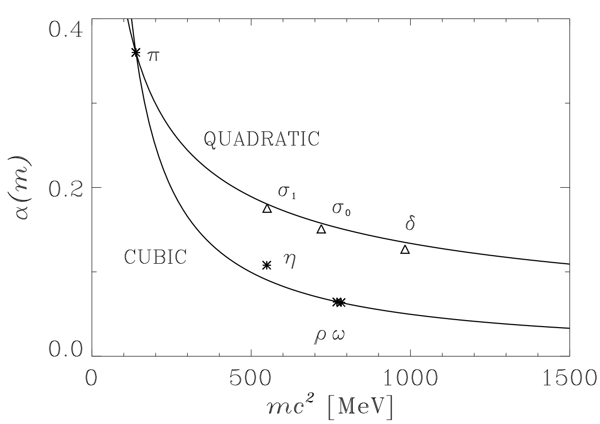

FIG. 3. Coupling constants α_β predicted by the scaling law (5.2) for quadratic and cubic self-interaction compared to the values which were found by fitting the Bonn B amplitudes.

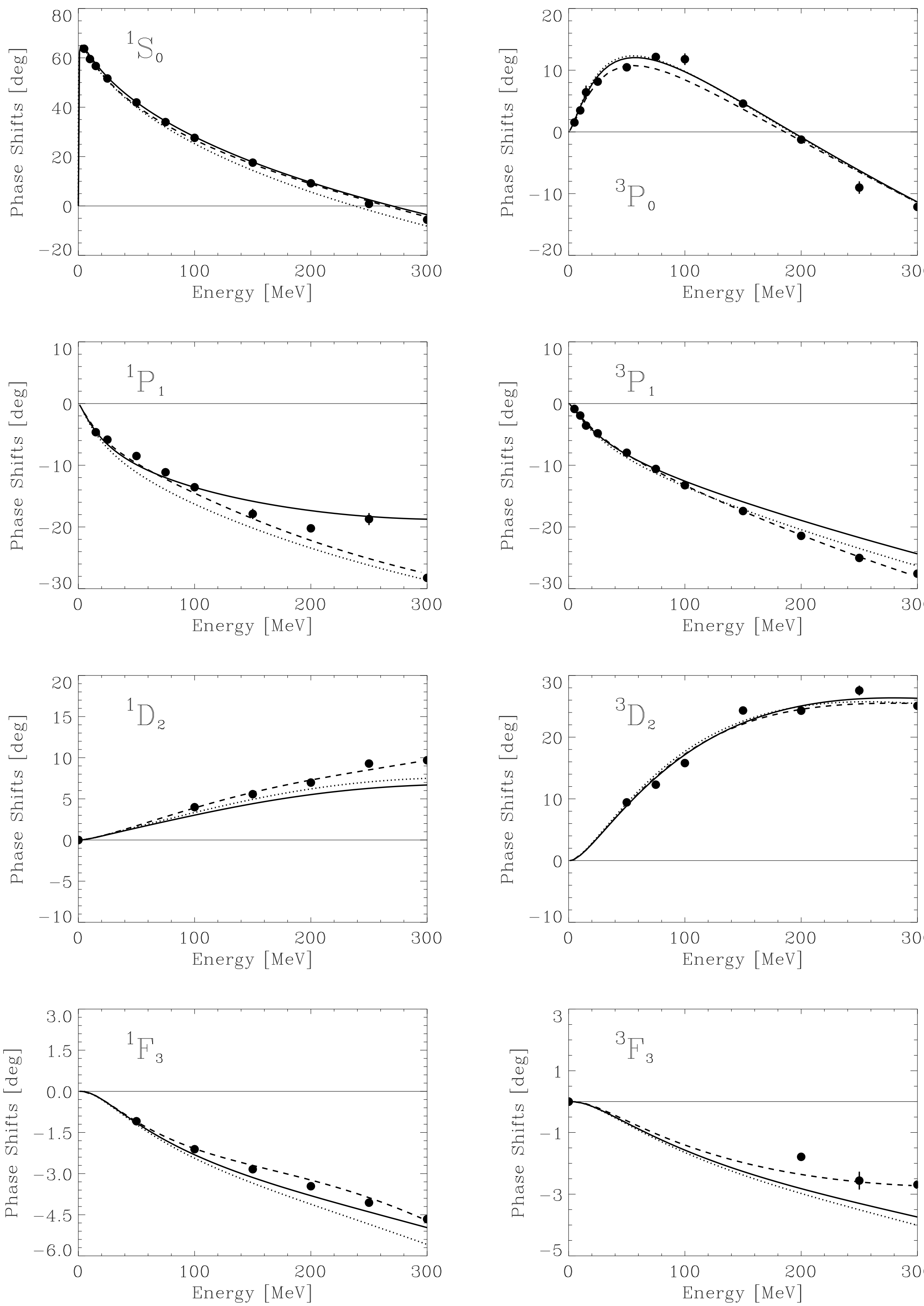

FIG. 4. *np* single channel phase shifts. We show Arndt SM95 (circles), Nijmegen PWA (dashed), Bonn B (dotted) and OSBEP (full).

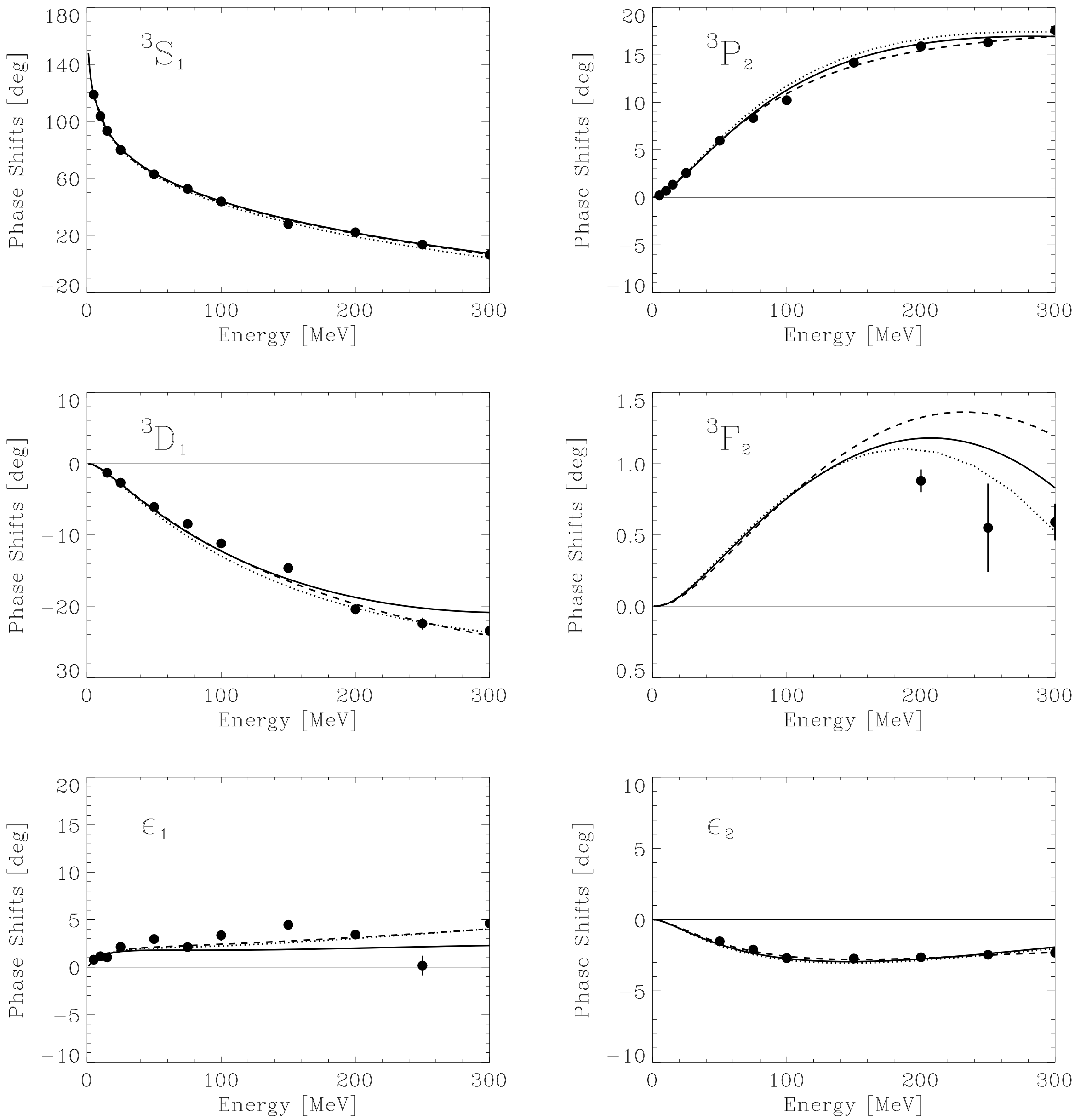

FIG. 5. SYM Phase shifts for the coupled $^3SD_1$ and $^3PF_2$ channel.